



## Decision Support

## Weighted Multivariate Mean Square Error for processes optimization: A case study on flux-cored arc welding for stainless steel claddings

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## ABSTRACT

A mathematical programming technique developed recently that optimizes multiple correlated characteristics is the Multivariate Mean Square Error (MMSE). The MMSE approach has obtained noteworthy results, by avoiding the production of inappropriate optimal points that can occur when a method fails to take into account a correlation structure. Where the MMSE approach is deficient, however, is in cases where the multiple correlated characteristics need to be optimized with varying degrees of importance. The MMSE approach, in treating all responses as having the same importance, is unable to attribute the desired weights. This paper thus introduces a strategy that weights the responses in the MMSE approach. The method, called the Weighted Multivariate Mean Square Error (WMMSE), utilizes a weighting procedure that integrates Principal Component Analysis (PCA) and Response Surface Methodology (RSM). In doing so, WMMSE obtains uncorrelated weighted objective functions from the original responses. After being mathematically programmed, these functions are optimized by employing optimization algorithms. We applied WMMSE to optimize a stainless steel cladding application executed via the flux-cored arc welding (FCAW) process. Four input parameters and eight response variables were considered. Stainless steel cladding, which carries potential benefits for a variety of industries, takes low cost materials and deposits over their surfaces materials having anti-corrosive properties. Optimal results were confirmed, which ensured the deposition of claddings with defect-free beads exhibiting the desired geometry and demonstrating good productivity indexes.

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## 1. Introduction

The literature offers several techniques for multi-objective optimizations. Among those that have been used in different applications are the desirability function (Derringer and Suich, 1980), the generalized distance (Khuri and Conlon, 1981), the multivariate integration (Chiao and Hamada, 2001), the multivariate yield with Gaussian Quadrature Reduction (Liu et al., 2002), the preemptive nonlinear goal programming (Kovach and Cho, 2009) and more recently the robust desirability function that considers model uncertainty (He et al., 2012). Many of these methods, however, are unconcerned with a common phenomenon that occurs in the modeling of manufacturing processes – the correlation among the multiple responses (Huang and Lin, 2008; Paiva et al., 2007; Safizadeh, 2002). The presence of such correlation, according to Box et al. (1973), can influence the optimization results. This destabilizes the mathematical models and produces errors in the regression coefficients. Consequently, if the variance–covariance (or correlation) structure is ignored, the regression equations

cannot adequately represent the objective or constraint functions (Chiao and Hamada, 2001; Khuri and Conlon, 1981).

Concerned with these issues, Paiva et al. (2009) proposed an optimization method that combines the Principal Component Analysis (PCA) and the Response Surface Methodology (RSM), converting the original multiple correlated objective functions in a new set of uncorrelated ones, while considering the respective targets. Nevertheless, if the multiple responses introduce different degrees of importance, this approach—though capable of eliminating the correlation's effect—is unable to attribute the desired weights. This inability stems from the correlation matrix being unable to transfer to the components the weights assigned to original responses.

Addressing therefore this gap in the work of Paiva et al. (2009), this paper's objective is to put forward the Weighted Multivariate Mean Square Error (WMMSE). The WMMSE is a method developed for the optimization of multiple correlated responses that present varying degrees of importance. In the multivariate approach, the weights attribution cannot be performed, as it can in conventional techniques, directly in the objective functions. A step-by-step procedure was thus developed for this very purpose. To model and optimize the problem, the WMMSE combines, similar to the method proposed by Paiva et al. (2009), the PCA, the RSM, and the concept of MSE (Mean Square Error).

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To demonstrate its applicability, we employed the WMMSE to optimize the cladding process of depositing AISI 316L stainless steel onto AISI 1020 carbon steel plates using flux-cored arc welding (FCAW). Four input parameters configured the FCAW process and to describe the cladding application we relied on eight characteristics.

In welding, depositing a layer of filler metal over the surface of another material to obtain desired properties or dimensions describes a process known as cladding. Cladding can serve three purposes: (1) to extend the useful life of a part that, for a given application, lacks needed properties; (2) to restore elements affected by corrosion or wear; and (3) to create surfaces with special features (Phillips, 1965). In industrial settings, the last of these three applications has been noticeably on the rise, especially so regarding stainless steel claddings (Palani and Murugan, 2007; Murugan and Parmar, 1997). Thus, by depositing stainless steel over other materials, such as carbon steels or low-alloy steels, manufacturers are able to turn out anti-corrosive surfaces. The process has proven attractive based on its impressive results – transforming common and low cost materials into corrosion-resistant surfaces.

The procedure, however, is complex. As such, it must of course achieve this application's peculiarities: dilution control and a weld bead with the desired geometric profile. Beyond this, the procedure must yield high quality and satisfactory productivity levels. So, the challenge for engineers is to control and optimize, after introducing several welding input parameters and multiple response variables, the stainless steel cladding process.

## 2. Theoretical fundamentals

### 2.1. Multivariate Mean Square Error

In the context of Robust Parameter Design (RPD) optimization (Montgomery, 2009), the Mean Square Error (MSE) is an objective function that combines the response surfaces of mean  $\hat{y}(\mathbf{x})$  and variance  $\hat{\sigma}^2(\mathbf{x})$  besides the respective target ( $T$ ). This objective function is subjected only to the experimental region constraint, as first suggested by Lin and Tu (1995):

$$\min_{\mathbf{x} \in \Omega} \text{MSE} = [\hat{y}(\mathbf{x}) - T]^2 + \hat{\sigma}^2(\mathbf{x}) \quad (1)$$

In Eq. (1),  $\mathbf{x}$  represents the vector of controllable variables or input parameters and  $\Omega$  denotes the experimental region in which  $\mathbf{x}$  is inserted. Therefore, the minimization of this objective function promotes the product's quality improvement. This expression refers to the mean and variance of only one response surface. For the multiple response case, Köksoy (2006) proposed the agglutination of several MSE functions, which could be either weighted or not. Then, if different degrees of importance are desired, the global objective function can be written as:

$$\text{MSE}_T = \sum_{i=1}^p w_i \cdot \text{MSE}_i = \sum_{i=1}^p w_i \cdot \left\{ [\hat{y}_i(\mathbf{x}) - T_i]^2 + \hat{\sigma}_i^2(\mathbf{x}) \right\} \quad (2)$$

where  $\text{MSE}_T$  is the global Mean Square Error,  $p$  is the number of response surfaces and  $w_i$  are the desired weights.

Also available are other multi-objective optimization routines that consider the response targets, such as the metrics Lp proposed by Ardakani and Noorossana (2008):

$$\begin{aligned} \min \bar{f}(\mathbf{x}) &= \sum_{i=1}^p w_i \cdot \left[ \frac{f_i(\mathbf{x}) - f_i^l}{f_i^{\text{MAX}} - f_i^l} \right]^2 \\ \text{s.t. : } \mathbf{x}^T & \\ \mathbf{x} &\leq \rho^2 \quad 0 \leq w_i \leq 1 \end{aligned} \quad (3)$$

In the formulation of Eq. (3),  $\bar{f}(\mathbf{x})$  is the global objective function and the values  $f_i^l$  and  $f_i^{\text{MAX}}$  are obtained from the payoff matrix of the objective functions, where  $f_i^l$  represents the found value with the individual optimization of  $f_i(\mathbf{x})$  and  $f_i^{\text{MAX}}$  is the maximum value observed for the  $i$ th objective function. The expression  $\mathbf{x}^T \mathbf{x} \leq \rho^2$  describes the constraint for a spherical experimental region, where  $\rho$  is the sphere's radius.

All the aforementioned expressions share the same drawback: they take no account of the correlation's influence on the optimization results. To consider the correlation, Govindaluri and Cho (2007) presented the following formulation:

$$\begin{aligned} \text{MSE}_i &= (\hat{y}_i(\mathbf{x}) - T_i)^2 + \hat{\sigma}_i^2(\mathbf{x}) + \sum_{j=1}^{i-1} \frac{\hat{\sigma}_i(\mathbf{x})}{\hat{\sigma}_i(\mathbf{x}) + \hat{\sigma}_j(\mathbf{x})} \cdot \\ &\quad [\hat{\sigma}_{ij}(\mathbf{x}) + (\hat{y}_i(\mathbf{x}) - T_i) \cdot (\hat{y}_j(\mathbf{x}) - T_j)] \end{aligned} \quad (4)$$

Although coherent, the computation of covariance response surface  $\hat{\sigma}_{ij}(\mathbf{x})$  in this proposal is only possible if the practitioner has a replicate or a crossed array, which—and here is the main drawback of this proposal—increases substantially the number of experiments.

Also considering that the correlation structure may significantly jeopardize the optimization results, Vining (1998) presented a minimization of a multivariate expected loss function as a multi-objective function:

$$E[L[y(\mathbf{x}), \theta]] = [E[y(\mathbf{x})] - \theta]^T \mathbf{C} [E[y(\mathbf{x})] - \theta] + \text{trace}[\mathbf{C} \sum_y(\mathbf{x})] \quad (5)$$

where  $E[L[y(\mathbf{x}), \theta]]$  is the multivariate expected loss function,  $\mathbf{x}$  represents the vector of controllable variables,  $y(\mathbf{x})$  is the vector of quality characteristics,  $\mathbf{C}$  is a  $p \times p$  positively defined matrix of costs (or weights) associated with the losses incurred when  $y(\mathbf{x})$  deviates from their respective targets,  $p$  is the number of quality characteristics, and  $\Sigma_y$  is the variance–covariance matrix. Likewise, Chiao and Hamada (2001) proposed a multivariate integration approach as a correlated multi-response optimization method. Using a specified region of responses, the optimal solution disregards any consideration of the targets. This formulation is written as:

$$\begin{aligned} \text{Max } P(Y \in S) &= \frac{1}{\sqrt{|\Sigma|} (2\pi)^p} \left[ \int_{a_1}^{b_1} \int_{a_2}^{b_2} \dots \int_{a_p}^{b_p} e^{-\frac{1}{2}(Y-\mu)^T \Sigma^{-1} (Y-\mu)} dY \right] \\ \text{s.t. : } \mathbf{x}^T \mathbf{x} &\leq \rho^2 \end{aligned} \quad (6)$$

where  $Y$  is the vector of multiple responses,  $S$  is the specified region for all responses formed by the lower bounds  $a_i$  and upper bounds  $b_i$ ,  $\Sigma$  is a  $p \times p$  positive definite variance–covariance matrix (with  $p$  the number of responses), and  $\mathbf{x}^T \mathbf{x} \leq \rho^2$  denotes a constraint for a spherical experimental region. Although efficient, the computation of a multivariate normal integral is no trivial task.

Trying to overcome the drawbacks of the correlation's negligence, Bratchell (1989) proposed using a second-order response surface based on PCA. Doing so helped to adequately represent the original set of responses in a small number of latent variables. Innovative though it was, Bratchell's approach offered no alternatives in cases where the largest principal component was unable to explain the greatest part of variance. Furthermore, it gave no indication of how the specification limits and targets of each response could be transformed to the plane of principal components.

Departing from Bratchell's idea, Paiva et al. (2009) combined the concept of MSE functions with response surfaces written in terms of principal components scores. The result of this combination also considered the original targets transformed into scores. Bratchell's approach considered only the first principal component; Paiva's method included a geometric mean of principal

component regressions whose the cumulative explanation exceeded 80%. Paiva et al. (2009) called this approach the Multivariate Mean Square Error (MMSE) (Paiva et al., 2012, 2009). The MMSE method begins with PCA converting a data set of correlated responses into new uncorrelated variables—termed principal components. These principal component scores are then developed, through RSM, into response surface functions. Finally, considering the MSE formulation, the estimated mean  $\hat{y}(\mathbf{x})$  is replaced by the principal component function  $PC$ . Since the  $i$ th eigenvalue is the variance of the  $i$ th principal component (Johnson and Wichern, 2007), the variance  $\hat{\sigma}^2(\mathbf{x})$  can be replaced by the eigenvalue  $\lambda$  and the target  $T$  is transformed into the principal component target,  $\zeta_{PC}$ . The MMSE is then stated by the following expression:

$$MMSE = (PC - \zeta_{PC})^2 + \lambda \quad (7)$$

In Eq. (1),  $PC$  is a second-order polynomial positioned in relation to the input variables. The principal component target ( $\zeta_{PC}$ ) is defined heuristically. According to Johnson and Wichern (2007), the  $i$ th principal component score is the product of the standardized values  $Z(\cdot)$  of the original responses  $Y_j$ , multiplied by its respective eigenvectors  $e_j$ . In this case, the standardized normal variable is calculated considering the mean, such as  $Z(Y_j|\mu_{Y_j})$ . Then, it becomes straightforward that if we consider the target established for the  $j$ th response,  $T_{Y_j}$ , the target written in terms of the principal component ( $\zeta_{PC}$ ) is established by:

$$\zeta_{PC} = e^T [Z(Y_j|T_{Y_j})] = \sum_{j=1}^p e_j \cdot [Z(Y_j|T_{Y_j})] \quad (8)$$

where  $Z(Y_j|T_{Y_j}) = (T_{Y_j} - \mu_{Y_j}) \cdot (\sigma_{Y_j})^{-1}$ ,  $\mu_{Y_j}$  is the mean of the  $j$ th response and  $\sigma_{Y_j}$  is the standard deviation of the  $j$ th response.

In this method, the optimization is given by minimizing the  $MMSE$  stated in Eq. (1), which means that the principal component tends to reach the established target with minimum variance. If more than one principal component is needed, then the MMSE optimization is obtained by the following mathematical formulation:

$$\begin{aligned} \text{Min } MMSE_T &= \left[ \prod_{i=1}^m MMSE_i \right]^{\left(\frac{1}{m}\right)} = \left\{ \prod_{i=1}^m [(PC_i - \zeta_{PC_i})^2 + \lambda_i] \right\}^{\left(\frac{1}{m}\right)}, \quad m \leq p \\ \text{s.t.: } g_n(\mathbf{x}) &\leq 0 \end{aligned} \quad (9)$$

where  $MMSE_T$  is the Total Multivariate Mean Square Error,  $MMSE_i$  is the Multivariate Mean Square Error for the  $i$ th principal component,  $m$  the number of needed principal components,  $p$  the number of responses,  $PC_i$  the response surface function for the  $i$ th principal component,  $\zeta_{PC_i}$  the target for the  $i$ th principal component,  $\lambda_i$  the eigenvalue for the  $i$ th principal component, and  $g_n(\mathbf{x}) \leq 0$  are the constraint equations.

The optimization of the principal components implies the optimization of the original problem. On this basis, engineers have characterized the MMSE approach as an intriguing method. While it can deal with the correlation among multiple responses that present different degrees of importance, the MMSE approach is unable to allocate weights to those responses. To answer such a deficiency, the Weighted Multivariate Mean Square Error (WMMSE) was developed. The WMMSE method is a way of optimizing multiple responses that are correlated but present varying degrees of importance. It is described in greater detail in the next section.

## 2.2. Weighted Multivariate Mean Square Error

Unlike traditional approaches, the MMSE approach cannot, in the objective function, directly attribute weights. This is because

the  $MMSE_T$  function is written in terms of principal components. In this function then, directly attributing weights does not mean that the original responses are weighted. The approach simply weighs the principal components, failing to ensure the weighting of the responses. The WMMSE proposes then that before constructing the objective function and before carrying out the Principal Component Analysis the weights be attributed. To do so, we developed a step-by-step procedure:

- Step 1: Standardize the data set of the original correlated responses using the transformation  $Z(Y_j|\mu_{Y_j}) = (y - \mu_{Y_j}) \cdot (\sigma_{Y_j})^{-1}$ .
- Step 2: Multiply each standardized response by its respective weight  $w_j$ , such that  $\sum w_j = 1$ .
- Step 3: Perform the Principal Component Analysis on the standardized and weighted responses using the variance–covariance matrix (unlike the MMSE approach, in which the PCA is performed using the correlation matrix).
- Step 4: Define the number of principal components that must be retained in the analysis and keep their respective eigenvalues ( $\lambda_i^*$ ), eigenvectors ( $e_i^*$ ) and scores.
- Step 5: Using the scores obtained in Step 4, establish RSM models for the significant principal components.
- Step 6: Calculate the respective targets for each principal component taking into account the weighted eigenvectors.
- Step 7: Apply the WMMSE formulation.

Step 1, the standardization of the responses, is important because it unifies the data set. Multiple responses, in practice, present different magnitudes and measurement units. Weighting the responses, Step 2, can be done by several ways. We employed Ch'ng et al.'s strategy (2005), which establishes that the total sum of the weights must equal one. Thus, one can easily determine the percentage of the total weight attributed to each response. Step 3 presents the relevant difference between the MMSE and WMMSE approaches. The MMSE performs PCA using the correlation matrix; the WMMSE does so using the variance–covariance matrix. Such a change is necessary because the variance–covariance matrix is able to assign to the principal components the weights attributed in Step 2. Such assigning is what the correlation matrix is unable to do. Finally, the next steps (Steps 4–6) are similar to those in the MMSE method. The difference is, however, they are executed taking into account the weighted responses.

As the WMMSE formulation was developed (Step 7), the multiplier  $\left[ \prod_{i=1}^m (\cdot) \right]$  was found to be an inappropriate choice for compounding the agglutination function. This operator can decouple the weights of the responses and thereby create a unique constant. Given this possibility, we replaced it with the sum  $\left( \sum_{i=1}^m (\cdot) \right)$  and also established a weighting criterion for the principal components. These were based on the respective degrees of explanation of each component.

Therefore, the WMMSE method obtains the optimization of multiple correlated responses presenting different degrees of importance through the following formulation:

$$\begin{aligned} \text{Min } WMMSE_T &= \sum_{i=1}^m \left[ \frac{v_i}{v_T} \cdot WMMSE_i \right] = \sum_{i=1}^m \left\{ \frac{v_i}{v_T} \cdot \left[ (PC_i^* - \zeta_{PC_i}^*)^2 + \lambda_i^* \right] \right\}, \quad m \leq p \\ \text{s.t.: } g_n(\mathbf{x}) &\leq 0 \end{aligned} \quad (10)$$

where  $WMMSE_T$  is the Total Weighted Multivariate Mean Square Error,  $WMMSE_i$  is the Weighted Multivariate Mean Square Error for the  $i$ th principal component,  $m$  the number of needed principal components,  $p$  the number of responses,  $v_i$  the degree of explanation for the  $i$ th principal component, such that  $\sum v_i = v_T$ ,  $PC_i^*$  is the response surface function for the  $i$ th principal component

obtained with the weighted responses,  $\bar{z}_{PC_i}^w$  the target for the  $i$ th principal component obtained with the weighted responses,  $\lambda_i^*$  the eigenvalue for the  $i$ th principal component obtained with the weighted responses, and  $g_n(\mathbf{x}) \leq 0$  are the constraint equations.

In both the MMSE and WMMSE approaches, the optimal points are identified by using the optimization algorithms on the respective presented formulations. To carry out this task, this work used the Genetic Algorithm (GA). The GA, broadly employed to optimize processes and operations, is considered by a number of researchers to be effective at searching for global solutions (Georgieva and Jordanov, 2009; Poojari and Varghese, 2008; Busacca et al., 2001).

### 2.3. The stainless steel cladding process

The stainless steel cladding process deposits a stainless steel layer on surfaces of carbon steel or low-alloy steels. This produces claddings with anti-corrosive properties and resistance that help surfaces withstand environments subject to high wear due to corrosion (Palani and Murugan, 2006). Since the base metal can be a common material, stainless steel cladding is dramatically less expensive than components made purely from high-priced stainless steel. Thus, carbon (or low-alloy) steel clad with stainless steel has been employed by such industries as the petroleum, chemical, food, agricultural, nuclear, naval, railway, civil construction and others (Kannan and Murugan, 2006; Murugan and Parmar, 1994).

How the cladding process mainly differs from conventional welding, as depicted in Fig. 1, is in the weld bead geometry. To ensure weld resistance (Fig. 1a), typical applications require high penetration ( $P$ ). In cladding applications, however, the desired weld bead geometry is characterized by broad bead width ( $W$ ), high reinforcement ( $R$ ), low penetration ( $P$ ) and low dilution percentage ( $D$ ) (Fig. 1b). This geometric profile is important; it ensures

the process covers the largest possible area with the least number of passes, saving significant material and time.

Just as important as the weld bead geometry is the dilution control. Dilution control, according to several researchers, is a critical aspect that establishes the final quality of the claddings (Kannan and Murugan, 2006; Palani and Murugan, 2006; Balasubramanian et al., 2009b). Shahi and Pandey (2008b) argue that the dilution strongly influences the chemical composition and properties of clad components; high dilutions increase the diffusion of elements between the filler and base metal (Fig. 2). So for stainless steel claddings, such an increase in the diffusion of elements can, in the clad layer, reduce the alloying elements and increase the carbon content—qualities that give rise to a number of metallurgical problems.

The previous considerations show that manufacturers, to achieve the desired quality of stainless steel cladding, must manage several characteristics. The control of this process, however, is not limited to geometric aspects. Manufacturers must also consider variables related to productivity and surface finishing; the exigency of these factors is always intense. Therefore, to identify a global condition that satisfies all these requirements, manufacturers must work out, employing specific techniques, the control and optimization of the stainless steel cladding process.

The literature reflects arising interest in stainless steel cladding applications. Most of these studies, however, are concerned with the analysis of the final properties of the claddings (Anjos et al., 1997; Majumdar et al., 2005; Kuo et al., 2009) or with the mathematical modeling of the process (Kannan and Murugan, 2006; Palani and Murugan, 2006; Shahi and Pandey, 2008b). Few researchers have worked with its optimization (Palani and Murugan, 2007; Balasubramanian et al., 2009a). Also scarce are works considering the analysis of variables related to the process productivity (Shahi and Pandey, 2008a; Tarnig et al., 2002).

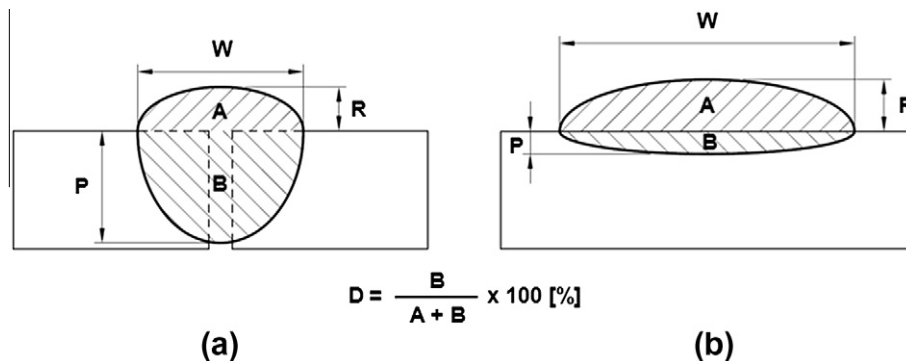


Fig. 1. Desired weld bead geometry: (a) typical welding bead, (b) cladding.

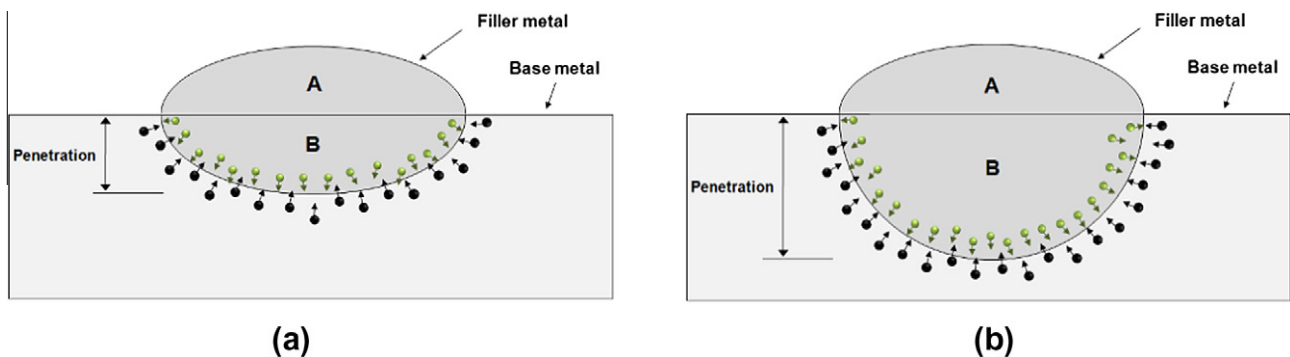


Fig. 2. Effect of the increase in dilution: (a) low dilution, low diffusion of elements, (b) high dilution, high diffusion of elements.

### 3. Experimental method

To apply the WMMSE method to optimizing stainless steel cladding, the research method, based on experiments, was conducted in three stages. One initial unknown was the behavior of flux-cored arc welding on the cladding characteristics. Hence, in the first stage, we employed, the Response Surface Methodology (RSM) to determine the objective functions for the original responses. RSM is defined by Montgomery (2009) as a collection of mathematical and statistical techniques useful in modeling and analyzing processes. Such an approach was followed in planning the experiments, collecting the data, and modeling the responses of interest. In the second stage, the correlation structure among the responses was analyzed and the process optimized by the MMSE method, thereby obtaining a primary solution; it characterized the necessity of optimizing the multiple responses with different weights and thus concluded the second stage. The third and final stage was the applying of the proposed WMMSE optimization, identifying a new optimal point from the attributed weights.

#### 3.1. Experiment planning

The FCAW parameters defined as input variables were the wire feed rate, voltage, welding speed, and distance of contact tip from work piece. The schedule of experiments was based on a central composite design (CCD), created for four parameters at two levels ( $2^k = 2^4 = 16$ ), eight axial points ( $2k = 8$ ), and seven center points. This resulted in 31 experiments. To specify the parameter levels, previous studies and preliminary tests were taken into account. Thus, after analyzing the previous works, the limits of each variable were pre-fixed and the preliminary tests were performed to verify the process behavior on the extreme conditions. At the end of this analysis, the parameter levels were fixed, as shown in Table 1. In the CCD matrix, a coded distance  $\alpha$  of 2.0 was adopted for the center points to the axial points.

The set of responses included eight welding outputs, six of them to be optimized and two to be taken as constraints. Those to be optimized included four that described the weld bead geometry – the bead width ( $W$ ), penetration ( $P$ ), reinforcement ( $R$ ) and dilution ( $D$ ). The remaining two described the process productivity – the deposition rate ( $DR$ ) and process yield ( $Y$ ). The two outputs taken as constraints concerned the surface quality – slag formation ( $SF$ ) and surface appearance ( $SA$ ). This was to ensure the deposition of optimal claddings with defect-free beads.

#### 3.2. Experimental procedure

To perform the experiments, the equipment used included a welding machine ESAB AristoPower 460, a module AristoFeed 30–4 watt MA6 (employed to feed the wire), and a mechanical system device. The latter was used to control the welding speed and the torch position (distance and angle). The base metal was carbon

**Table 2**

Chemical composition of base metal and filler metal.

| Material        | C             | Mn            | P    | S    | Si   | Ni   | Cr   | Mo   |
|-----------------|---------------|---------------|------|------|------|------|------|------|
| AISI 1020       | 0.18/<br>0.23 | 0.30/<br>0.60 | 0.04 | 0.05 | –    | –    | –    | –    |
| AWS E316LT1-1/4 | 0.03          | 1.58          | –    | –    | 1.00 | 12.4 | 18.5 | 2.46 |

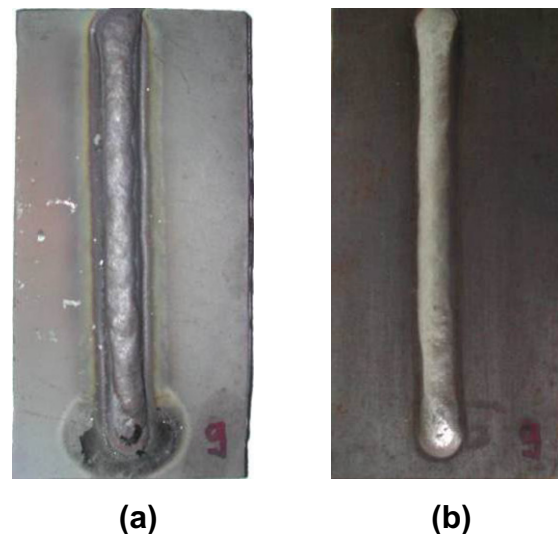
steel AISI 1020, cut into plates of  $120 \times 60 \times 6.35$  millimeter. The filler metal employed was a flux-cored stainless steel wire type AWS E316LT1-1/4, with a diameter of 1.2 millimeter. Table 2 presents the chemical composition of these materials.

For the welding technique, the experiments were performed by simply depositing a stainless steel bead on carbon steel plates (bead on plate), taking into account the parameters defined by the CCD matrix. The shielding gas used was a mixture of 75% Ar + 25% CO<sub>2</sub> at a flow rate of 16 liter/minutes. The torch angle was set at 15° to “pushing.”

To record the responses, initially the surface quality was evaluated through scores assigned by the researchers. Thus, for slag formation, the scores ranged from 1 to 5, with 1 signifying the worst slag formation and 5 the best; the best slag formation should have a complete coating on the weld. Next, the slag was removed and the surface appearance—the presence or absence of defects—observed. For this response, the scores ranged from 1 to 10, with 1 signifying an entirely defective weld bead and 10 a flawless one. An example of this evaluation is shown in Fig. 3.

After recording the quality responses, the deposition rate and the process yield (productivity responses) were calculated. These calculations took into account the welding time and masses of the carbon steel plates observed before and after the deposition of beads.

The weld bead geometry was measured at four points along the specimens. The beginning and end of the process were discarded and an average of the responses was recorded. The samples were cut and their cross sections were properly prepared, attacked with 4% nital, and photographed. The image analysis software Analysis Doc® was utilized to measure the weld bead dimensions, obtaining the bead width, penetration, reinforcement, penetration area, and total area of the weld. The dilution percentage was then calculated by dividing the penetration area by the total area. Fig. 4 illustrates



**Fig. 3.** Evaluation of the surface quality: (a) slag formation (score of 4), (b) surface appearance (score of 9).

**Table 1**  
Parameters and their levels.

| Parameters                               | Unit                   | Notation | Levels |      |      |      |      |
|------------------------------------------|------------------------|----------|--------|------|------|------|------|
|                                          |                        |          | –2     | –1   | 0    | +1   | +2   |
| Wire feed rate                           | meter/<br>minutes      | $W_f$    | 5.5    | 7.0  | 8.5  | 10.0 | 11.5 |
| Voltage                                  | volt                   | $V$      | 24.5   | 27.0 | 29.5 | 32.0 | 34.5 |
| Welding speed                            | centimeter/<br>minutes | $S$      | 20     | 30   | 40   | 50   | 60   |
| Contact tip to<br>work piece<br>distance | millimeter             | $N$      | 10     | 15   | 20   | 25   | 30   |

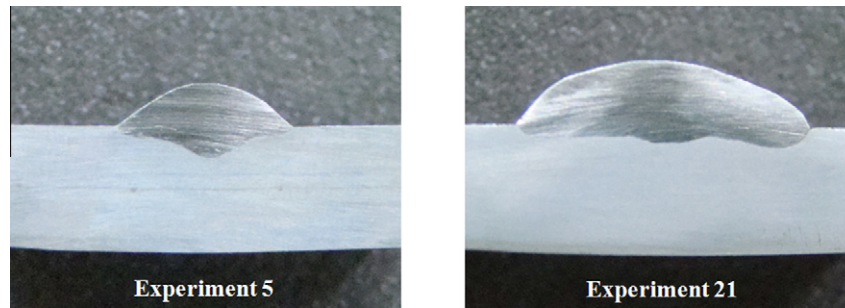


Fig. 4. Weld bead geometry after preparing the specimens.

Table 3

Experimental matrix.

| Run | Parameters         |          |                        |                | Geometry       |                |                |       | Productivity        |       | Quality    |            |
|-----|--------------------|----------|------------------------|----------------|----------------|----------------|----------------|-------|---------------------|-------|------------|------------|
|     | Wf (meter/minutes) | V (volt) | S (centimeter/minutes) | N (millimeter) | W (millimeter) | P (millimeter) | R (millimeter) | D (%) | DR (kilogram/hours) | Y (%) | SF (score) | SA (score) |
| 1   | −1                 | −1       | −1                     | −1             | 11.19          | 1.37           | 2.63           | 26.44 | 2.718               | 89.74 | 3          | 7          |
| 2   | 1                  | −1       | −1                     | −1             | 12.99          | 1.66           | 3.12           | 25.82 | 3.881               | 89.71 | 5          | 6          |
| 3   | −1                 | 1        | −1                     | −1             | 12.70          | 1.69           | 2.50           | 31.49 | 2.699               | 89.14 | 3          | 10         |
| 4   | 1                  | 1        | −1                     | −1             | 15.05          | 1.98           | 2.78           | 31.25 | 3.871               | 89.47 | 3          | *          |
| 5   | −1                 | −1       | 1                      | −1             | 9.21           | 1.65           | 2.17           | 36.22 | 2.773               | 91.58 | 3          | 10         |
| 6   | 1                  | −1       | 1                      | −1             | 9.96           | 1.94           | 2.67           | 33.69 | 3.924               | 90.70 | 4          | 9          |
| 7   | −1                 | 1        | 1                      | −1             | 9.75           | 1.54           | 2.06           | 37.12 | 2.647               | 87.43 | 3          | 10         |
| 8   | 1                  | 1        | 1                      | −1             | 11.51          | 2.18           | 2.42           | 41.08 | 3.822               | 88.36 | 3          | 8          |
| 9   | −1                 | −1       | −1                     | 1              | 10.32          | 1.25           | 2.87           | 22.46 | 2.740               | 90.49 | 4          | 9          |
| 10  | 1                  | −1       | −1                     | 1              | 11.43          | 1.00           | *              | 18.32 | 3.870               | 89.47 | 5          | 8          |
| 11  | −1                 | 1        | −1                     | 1              | 11.27          | 1.32           | 2.85           | 23.71 | 2.743               | 90.60 | 3          | 7          |
| 12  | 1                  | 1        | −1                     | 1              | 13.34          | 1.10           | 3.18           | 21.96 | 3.885               | 89.81 | 4          | 4          |
| 13  | −1                 | −1       | 1                      | 1              | 7.99           | 1.11           | 2.55           | 24.96 | 2.847               | 94.03 | 3          | 9          |
| 14  | 1                  | −1       | 1                      | 1              | 8.62           | 1.23           | 2.80           | 23.31 | 3.901               | 90.17 | 4          | 9          |
| 15  | −1                 | 1        | 1                      | 1              | 8.48           | 1.37           | 2.36           | 28.77 | 2.832               | 93.52 | 3          | 10         |
| 16  | 1                  | 1        | 1                      | 1              | 10.84          | 1.64           | 2.60           | 30.19 | 3.969               | 91.74 | 3          | 7          |
| 17  | −2                 | 0        | 0                      | 0              | 9.07           | 1.38           | 2.21           | 31.56 | 2.204               | 92.62 | 3          | 9          |
| 18  | 2                  | 0        | 0                      | 0              | 12.21          | 2.14           | 3.06           | 30.95 | 4.454               | 89.52 | 4          | 6          |
| 19  | 0                  | −2       | 0                      | 0              | 9.42           | 1.20           | 3.03           | 22.84 | 3.324               | 90.41 | 4          | 9          |
| 20  | 0                  | 2        | 0                      | 0              | 11.69          | 1.86           | 2.46           | 35.58 | 3.311               | 90.04 | 3          | 8          |
| 21  | 0                  | 0        | −2                     | 0              | 14.93          | 0.95           | *              | 18.58 | 3.319               | 90.27 | 4          | 8          |
| 22  | 0                  | 0        | 2                      | 0              | 8.48           | 1.43           | 2.25           | 35.78 | 3.423               | 93.08 | 3          | 9          |
| 23  | 0                  | 0        | 0                      | −2             | 11.73          | 2.18           | 2.61           | 40.44 | 3.242               | 88.15 | 3          | 8          |
| 24  | 0                  | 0        | 0                      | 2              | 9.22           | 1.28           | 2.89           | 24.16 | 3.385               | 92.05 | 3          | 8          |
| 25  | 0                  | 0        | 0                      | 0              | 10.82          | 1.71           | 2.60           | 31.05 | 3.421               | 93.04 | 3          | 8          |
| 26  | 0                  | 0        | 0                      | 0              | 10.93          | 1.72           | 2.59           | 31.67 | 3.380               | 91.91 | 3          | 8          |
| 27  | 0                  | 0        | 0                      | 0              | 10.74          | 1.62           | 2.65           | 30.88 | 3.402               | 92.51 | 3          | 7          |
| 28  | 0                  | 0        | 0                      | 0              | 10.61          | 1.80           | 2.50           | 32.83 | 3.382               | 91.98 | 3          | 8          |
| 29  | 0                  | 0        | 0                      | 0              | 10.64          | 1.49           | 2.62           | 29.99 | 3.388               | 92.15 | 3          | 7          |
| 30  | 0                  | 0        | 0                      | 0              | 10.59          | 1.49           | 2.61           | 31.09 | 3.398               | 92.40 | 3          | 7          |
| 31  | 0                  | 0        | 0                      | 0              | 10.57          | 1.50           | 2.56           | 31.02 | 3.404               | 92.58 | 3          | 8          |

the cross sections of the weld beads after the procedures of cutting, preparing, and attacking.

Once all the responses had been measured, they were assembled to compound the experimental matrix presented in Table 3 and used as a data source for the modeling and optimization of the process. Due to their being characterized as outliers, three data were discarded—two related to the reinforcement (Tests 10 and 21) and another to the surface appearance (Test 2).

## 4. Results and discussion

### 4.1. Modeling of the stainless steel cladding process

This study sought to determine the transfer functions among the welding input parameters and responses. Considering this and aiming to control the process during the optimization, we

developed response surface models for the eight cladding characteristics. According to RSM, if the experimental space is in a region of curvature, then the process is adjusted well by a second-order polynomial. The transfer function that relates a given response  $y$  with  $k$  input variables is described by the following expression (Montgomery, 2009):

$$\hat{y}(\mathbf{x}) = \beta_0 + \sum_{i=1}^k \beta_i x_i + \sum_{i=1}^k \beta_{ii} x_i^2 + \sum_{i < j} \beta_{ij} x_i x_j + \varepsilon \quad (11)$$

where  $\hat{y}(\mathbf{x})$  is the response of interest,  $x_i$  is the input parameters,  $\beta_0$ ,  $\beta_i$ ,  $\beta_{ii}$ ,  $\beta_{ij}$  are coefficients to be estimated,  $k$  is the number of input parameters and  $\varepsilon$  is the error observed in the response.

Therefore, writing Eq. (11) for the four welding parameters considered in this work, we can represent the stainless steel cladding responses as follows:



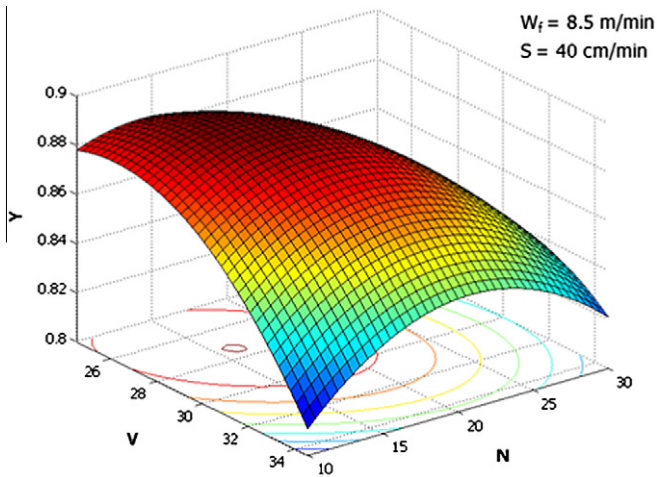


Fig. 6. Response surface graphic for process yield.

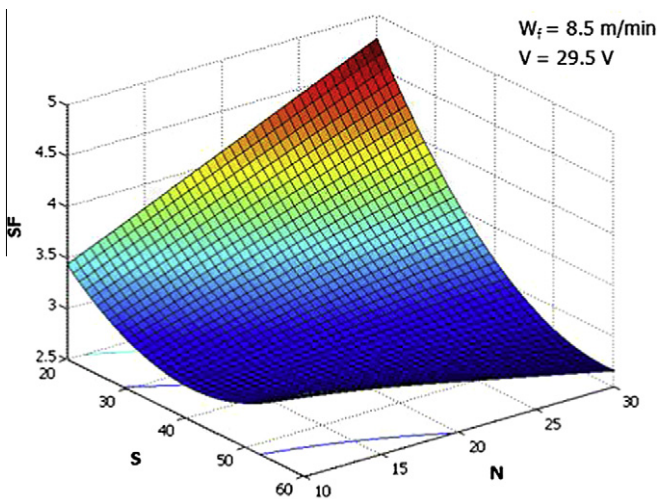


Fig. 7. Response surface graphic for slag formation.

the response surface graphics for some characteristics on which such analysis can be done.

#### 4.2. Optimization by the Multivariate Mean Square Error

To optimize the multiple correlated responses, we used the MMSE method. What was initially analyzed was resulting correlation structure among the responses to be optimized (Table 6). Since some significant relationships were identified, it can be seen that the cladding responses are moderately correlated. Using MMSE to optimize is thus justified.

Principal Component Analysis was next performed to find the uncorrelated principal components needed to represent the original responses. However, to perform PCA as a valid method, these variables must satisfy a multivariate normality assumption. Therefore the study carried out, as illustrated in Fig. 8, a multivariate normality test based on a chi-square quantile–quantile plot of the observations' squared Mahalanobis distances. Once the results showed a  $p$ -value equal to 1.000, with a significance level of 5%, the assumption of multivariate normality was tenable and PCA applicable. The PCA results (Table 7) indicated that three components were capable of representing 90.5% of the data. These three were used in the optimization.

**Table 6**  
Correlation among the responses.

|           | <i>W</i>                     | <i>P</i>                     | <i>R</i>                     | <i>D</i>        | <i>DR</i>       |
|-----------|------------------------------|------------------------------|------------------------------|-----------------|-----------------|
| <i>P</i>  | 0.220<br>0.234               |                              |                              |                 |                 |
| <i>R</i>  | <b>0.445</b><br><b>0.016</b> | −0.143<br>0.461              |                              |                 |                 |
| <i>D</i>  | −0.127<br>0.496              | <b>0.818</b><br><b>0.000</b> | −0.679<br><b>0.000</b>       |                 |                 |
| <i>DR</i> | <b>0.361</b><br><b>0.046</b> | 0.302<br>0.099               | <b>0.545</b><br><b>0.002</b> | −0.056<br>0.766 |                 |
| <i>Y</i>  | −0.533<br><b>0.002</b>       | −0.290<br>0.114              | −0.185<br>0.338              | −0.107<br>0.565 | −0.191<br>0.305 |

Cells: Pearson correlation.  $p$ -Value. Bold values have strong correlation.

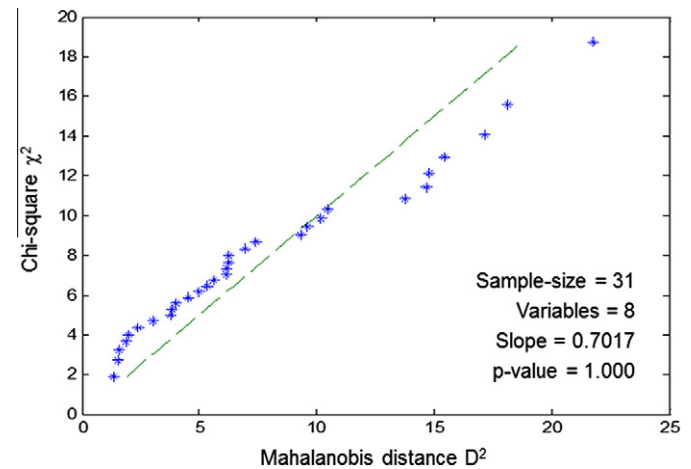


Fig. 8. Multivariate normality test.

**Table 7**  
Principal Component Analysis for the original responses.

|             |            |            |              |            |            |            |
|-------------|------------|------------|--------------|------------|------------|------------|
| Eigenvalue  | 2.492      | 2.081      | 0.865        | 0.388      | 0.144      | 0.038      |
| Proportion  | 0.415      | 0.347      | 0.143        | 0.065      | 0.024      | 0.006      |
| Cumulative  | 0.415      | 0.762      | <b>0.905</b> | 0.970      | 0.994      | 1.000      |
| Eigenvector | <i>PC1</i> | <i>PC2</i> | <i>PC3</i>   | <i>PC4</i> | <i>PC5</i> | <i>PC6</i> |
| <i>W</i>    | 0.506      | −0.208     | −0.267       | 0.705      | 0.365      | −0.010     |
| <i>P</i>    | 0.530      | 0.302      | 0.222        | 0.123      | −0.625     | 0.417      |
| <i>R</i>    | 0.167      | −0.650     | 0.049        | −0.034     | 0.516      | −0.530     |
| <i>D</i>    | 0.290      | 0.601      | 0.139        | −0.037     | 0.095      | −0.724     |
| <i>DR</i>   | 0.394      | −0.285     | 0.657        | −0.346     | 0.444      | 0.124      |
| <i>Y</i>    | −0.444     | 0.019      | 0.653        | 0.605      | −0.068     | −0.076     |

Data extracted from correlation matrix.

Bold value presents significant explanatory capacity.

The next step consisted of determining the quadratic models for the significant principal components. Thus, taking the scores calculated in PCA and modeling them according to RSM, the following expressions were obtained:

$$PC1 = -0.201 + 1.164W_f + 0.525V - 0.413S - 0.957N + 0.127W_f^2 + 0.064V^2 - 0.092S^2 + 0.194N^2 + 0.131W_fS + 0.096VS - 0.168VN - 0.200SN \quad (13)$$

$$PC2 = 0.139 - 0.801W_f + 0.586V + 1.237S - 0.938N - 0.185V^2 - 0.333S^2 + 0.184W_fV + 0.206W_fS - 0.115W_fN \quad (14)$$

$$\begin{aligned}
 PC3 = & 0.661 + 0.447W_f - 0.047V + 0.687S + 0.178N \\
 & - 0.165W_f^2 - 0.295V^2 - 0.291S^2 - 0.249N^2 \\
 & + 0.176W_fV - 0.346W_fN - 0.174VS + 0.302VN \\
 & + 0.274SN
 \end{aligned} \quad (15)$$

The ANOVA results for the PC models identified, for all of them,  $p$ -values of less than 5% significance. In relation to their adjustments, presented an  $adj. R^2$  of 96.34%, for 93.64%, and for 88.48%. Fig. 9 shows the normality test performed for the residuals of the PC models, characterizing all of them as normal variables.

The targets for the principal components were established based on the targets of the original responses. These latter ones were specified by the individual optimization of each response, using the developed RSM models. Thus, through Eq. (8) and using the data contained in Table 8, the targets for the principal components were calculated, resulting in  $-0.291$  for,  $-4.370$  for, and  $1.560$  for.

With the RSM models for the significant principal components developed and their respective targets calculated, the MMSE formulation could be put forward:

$$\begin{aligned}
 Min \ MMSE_T = & \{[(PC1 + 0.291)^2 + 2.492] \cdot [(PC2 + 4.370)^2 \\
 & + 2.081] \cdot [(PC3 - 1.560)^2 + 0.857]\}^{(4)} s.t. : \\
 & SF \geq 4 \ SA \geq 8 \ W_f^2 + V^2 + S^2 + N^2 \leq 4.000
 \end{aligned} \quad (16)$$

where  $PC1$   $PC2$   $PC3$  are the RSM models described in Eqs. (13)–(15); and are constraints for the surface quality, described by their

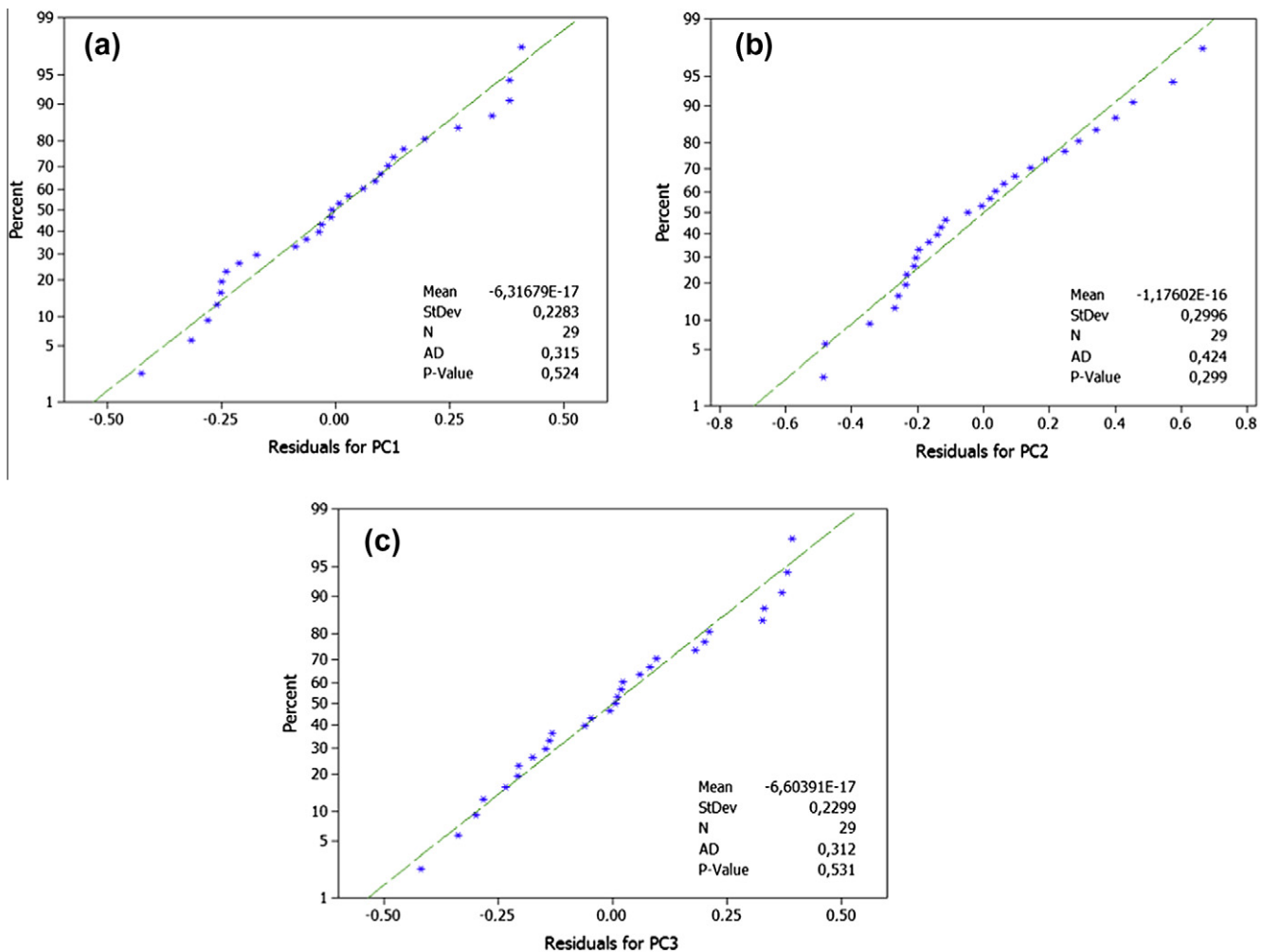
**Table 8**

Data used in the establishment of targets for the principal components.

|                    | $W$    | $P$    | $R$    | $D$    | $DR$   | $Y$    |
|--------------------|--------|--------|--------|--------|--------|--------|
| Mean               | 10.849 | 1.541  | 2.685  | 29.52  | 3.341  | 90.92  |
| Standard deviation | 1.735  | 0.334  | 0.350  | 5.86   | 0.511  | 1.67   |
| Target             | 15.574 | 0.827  | 3.341  | 16.27  | 4.456  | 94.90  |
| Objective          | Max    | Min    | Max    | Min    | Max    | Max    |
| Standardization    | 2.724  | −2.137 | 1.878  | −2.261 | 2.184  | 2.372  |
| Eigenvector $PC1$  | 0.506  | 0.530  | 0.167  | 0.290  | 0.394  | −0.444 |
| Eigenvector $PC2$  | −0.280 | 0.302  | −0.650 | 0.601  | −0.285 | 0.019  |
| Eigenvector $PC3$  | −0.267 | 0.222  | 0.049  | 0.139  | 0.657  | 0.653  |

respective RSM models; and  $W_f^2 + V^2 + S^2 + N^2 \leq 4.000$  is the spherical constraint for the experimental region, considering  $\rho = \alpha = 2.0$ .

To identify the optimal point, the genetic algorithm was applied in the previous formulation, programmed in a Microsoft Excel worksheet and employing the solver evolutionary supplement. Thus, considering 1,000 iterations, a convergence of 0.0001, a population size of 150 cases, and a mutation rate of 0.05, the optimal point was found. These results are presented in Table 9, obtained with a reliability of 95% for the following combination of welding parameters:  $W_f = 10.3$  meter/minutes,  $V = 27.0$  volt,  $S = 50.3$  centimeter/minutes, and  $N = 23.4$  millimeter.



**Fig. 9.** Residual analysis for PC models: (a) normality test for  $PC1$ ; (b) normality test for  $PC2$ ; (c) normality test for  $PC3$ .

**Table 9**

Optimal results for the stainless steel cladding process obtained with the MMSE method.

| Responses     | Geometry   |            |            |       | Productivity   |       | Quality |       |
|---------------|------------|------------|------------|-------|----------------|-------|---------|-------|
|               | W          | P          | R          | D     | DR             | Y     | SF      | SA    |
| Optimal point | 8.99       | 1.45       | 2.87       | 25.88 | 4.05           | 90.78 | 4       | 9     |
| Targets       | 15.57      | 0.83       | 3.34       | 16.27 | 4.46           | 94.9  | ≥4      | ≥8    |
| Units         | millimeter | millimeter | millimeter | %     | kilogram/hours | %     | score   | score |

A comparison of the optimized responses and the collected data in the experimental matrix classified the optimal weld bead geometry as unsatisfactory. It was verified that a broader bead width, a higher reinforcement, and a lower penetration could be reached. Furthermore, as can be seen in Table 9, the optimal results fell far short of the targets. To improve these results, we optimized the responses by attributing to them different weights. This was done by employing the Weighted Multivariate Mean Square Error.

#### 4.3. Optimization by the Weighted Multivariate Mean Square Error

The MMSE method optimized the weld bead geometry and the productivity using the same degrees of importance. Hence, the productivity optimization put the weld bead geometry optimization at a disadvantage. Therefore, the WMMSE was applied to a new optimization so as to improve these results by means of attributing different weights to the responses. The WMMSE method attributes weights differently than do conventional methods. In this part of the experiment, we developed the steps for how to do it.

##### Step 1 and Step 2: Standardization and weighting of the original responses

Aiming to unify the original data set, the responses were initially standardized by subtracting their means from each experimental value and, after, dividing them by their standard deviations. Then, each standardized response was multiplied by its respective weight. To attribute the weights, the weld bead geometry was considered twice as important as the productivity. Thus, the bead width, penetration, reinforcement, and dilution were multiplied by 0.2; the deposition rate and the process yield were multiplied by 0.1; thus, the total sum of the weights was 1.0.

##### Step 3: Principal Component Analysis

Once the original responses were standardized and weighted, Principal Component Analysis was performed on the data, taking into account the variance–covariance matrix. The results are presented in Table 10.

**Table 10**

Principal Component Analysis for the weighted responses.

| Eigenvalue  | 0.073  | 0.064  | 0.013        | 0.006  | 0.003  | 0.001  |
|-------------|--------|--------|--------------|--------|--------|--------|
| Proportion  | 0.453  | 0.399  | 0.082        | 0.039  | 0.020  | 0.008  |
| Cumulative  | 0.453  | 0.852  | <b>0.934</b> | 0.973  | 0.992  | 1.000  |
| Eigenvector | PC1*   | PC2*   | PC3*         | PC4*   | PC5*   | PC6*   |
| 0.2 · Z(W)  | 0.065  | 0.664  | 0.596        | −0.415 | 0.167  | 0.011  |
| 0.2 · Z(P)  | 0.535  | 0.403  | −0.353       | −0.004 | −0.534 | −0.376 |
| 0.2 · Z(R)  | −0.526 | 0.523  | −0.343       | 0.173  | −0.252 | 0.489  |
| 0.2 · Z(D)  | 0.652  | 0.000  | −0.100       | 0.062  | 0.212  | 0.719  |
| 0.1 · Z(DR) | −0.014 | 0.265  | −0.547       | −0.075 | 0.741  | −0.275 |
| 0.1 · Z(Y)  | −0.089 | −0.233 | −0.305       | −0.888 | −0.172 | 0.165  |

Data extracted from variance–covariance matrix.

Bold value presents significant explanatory capacity.

##### Step 4: Number of needed principal components

The PCA showed again that three principal components were needed to represent the responses, since they represented 93.4% of the data. These components were used in the optimization.

##### Step 5: RSM models for the significant principal components

The RSM models for the principal components of the weighted responses were developed similarly to those of the MMSE method. They are described by Eqs. (17)–(19). Again, all expressions presented *p*-values of less than 5% significance. The *adj.R*<sup>2</sup> values were 91.37% for, 97.62% for, and 89.86% for. Fig. 10 shows the residual analysis for these models, with normal results for all of them.

$$PC1^* = 0.065 - 0.036W_f + 0.144V + 0.186S - 0.228N \\ - 0.037V^2 - 0.070S^2 + 0.045W_fV + 0.046W_fS \\ - 0.035W_fN \quad (17)$$

$$PC2^* = -0.024 + 0.201W_f + 0.042V - 0.178S - 0.073N \\ + 0.014W_f^2 + 0.012V^2 + 0.023N^2 + 0.012W_fV \\ - 0.027W_fN \quad (18)$$

$$PC3^* = -0.053 - 0.066W_f + 0.040V - 0.090S - 0.024N \\ + 0.013V^2 + 0.044S^2 - 0.020W_fS + 0.040W_fN \\ - 0.020VN \quad (19)$$

##### Step 6: Targets for the principal components

The targets for the principal components of the weighted responses were calculated, also similarly to those of the MMSE method, using Eq. (8) and Table 11. It was found −3.671 for, 1.954 for, and 0.043 for.

##### Step 7: WMMSE formulation

Finally, after all the previous steps had been executed, the WMMSE formulation was given as:

$$\text{Min } WMMSE_T = \frac{0.453}{0.934} \cdot [(PC1^* + 3.671)^2 + 0.073] \\ + \frac{0.399}{0.934} \cdot [(PC2^* - 1.954)^2 + 0.064] + \frac{0.082}{0.934} \\ \cdot [(PC3^* - 0.043)^2 + 0.013] \\ s.t. : SF \geq 4 \\ SA \geq 8 \\ W_f^2 + V^2 + S^2 + N^2 \leq 4.000 \quad (20)$$

As presented in Section 2.3, it should be pointed out that in the WMMSE formulation the principal components are also weighted. However, these weights take into account the degrees of explanation observed for each component.

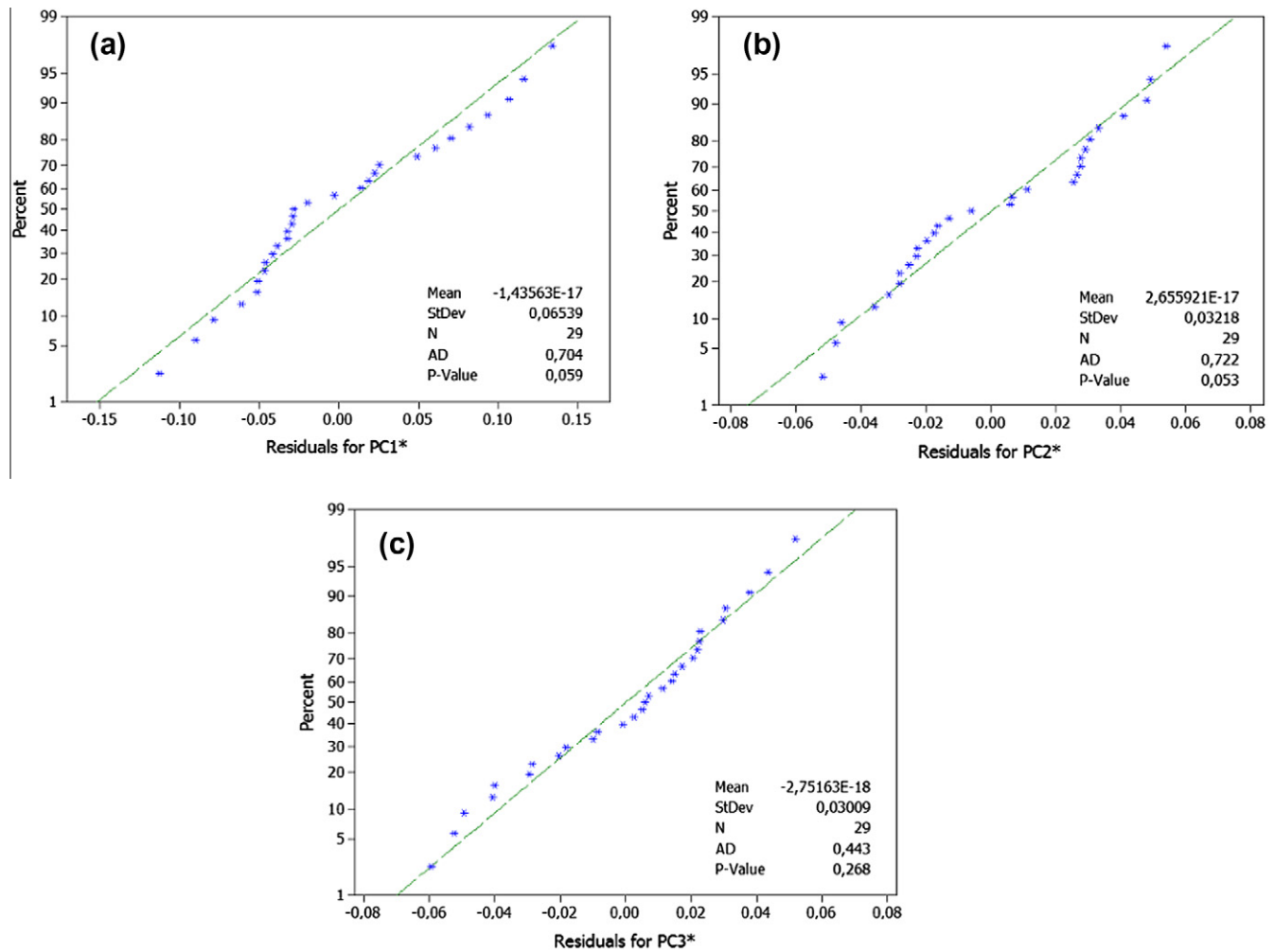


Fig. 10. Residual analysis for PC\* models: (a) normality test for PC1\*; (b) normality test for PC2\*; normality test for PC3\*.

Table 11

Data used in the establishment of targets for the principal components of the weighted responses.

|                    | W      | P      | R      | D      | DR     | Y      |
|--------------------|--------|--------|--------|--------|--------|--------|
| Mean               | 10.849 | 1.541  | 2.685  | 29.52  | 3.341  | 90.92  |
| Standard deviation | 1.735  | 0.334  | 0.350  | 5.86   | 0.511  | 1.67   |
| Target             | 15.574 | 0.827  | 3.341  | 16.27  | 4.456  | 94.90  |
| Objective          | Max    | Min    | Max    | Min    | Max    | Max    |
| Standardization    | 2.724  | -2.137 | 1.878  | -2.261 | 2.184  | 2.372  |
| Eigenvector PC1*   | 0.065  | 0.535  | -0.526 | 0.652  | -0.014 | -0.089 |
| Eigenvector PC2*   | 0.664  | 0.403  | 0.523  | 0.000  | 0.265  | -0.233 |
| Eigenvector PC3*   | 0.596  | -0.353 | -0.343 | -0.100 | -0.547 | -0.305 |

Table 12

Optimal results for the stainless steel cladding process obtained with the WMMSE method.

| Responses     | Geometry   |            |            |       | Productivity   |       | Quality |       |
|---------------|------------|------------|------------|-------|----------------|-------|---------|-------|
|               | W          | P          | R          | D     | DR             | Y     | SF      | SA    |
| Optimal point | 11.90      | 0.92       | 3.33       | 16.61 | 3.66           | 88.73 | 5       | 8     |
| Targets       | 15.57      | 0.83       | 3.34       | 16.27 | 4.46           | 94.9  | ≥4      | ≥8    |
| Units         | millimeter | millimeter | millimeter | %     | kilogram/hours | %     | score   | score |

The genetic algorithm was also applied in the WMMSE formulation, using the solver evolutionary supplement with the same search parameters. In this fashion, we optimized the flux-cored arc welding process for applications of AISI 316 stainless steel claddings deposited on AISI 1020 carbon steel plates. Table 12 presents the optimal results, obtained with 95% reliability for a  $W_f = 9.5$  - meter/minutes,  $V = 26.5$  volt,  $S = 27.3$  centimeter/minutes, and  $N = 23.6$  millimeter.

In terms of geometric profile, the WMMSE method outperformed the MMSE method; the WMMSE produced a broader bead width, a higher reinforcement, and a lower penetration. The difference can be observed in Fig. 11. Regarding the response targets, the penetration and dilution stayed close to their minimum values and the reinforcement practically reached its maximum value.

In terms of productivity, however, the WMMSE failed to outperform the MMSE method, a consequence of attributing to it a lower

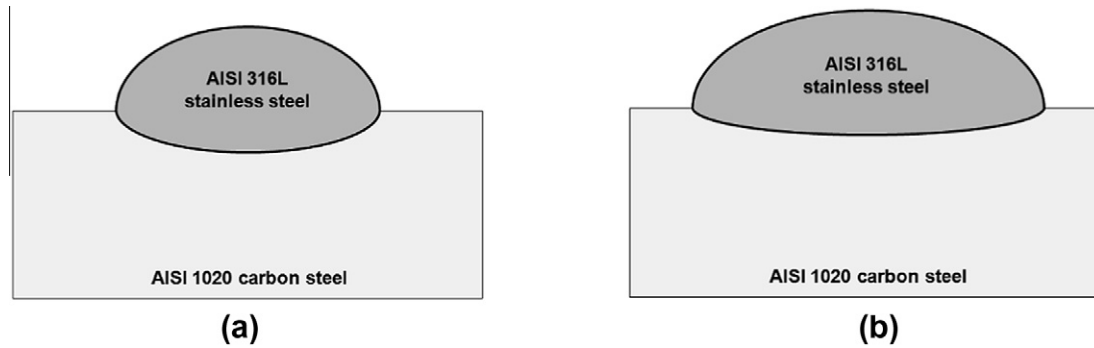


Fig. 11. Optimal weld bead geometries: (a) MMSE optimization, (b) WMMSE optimization.

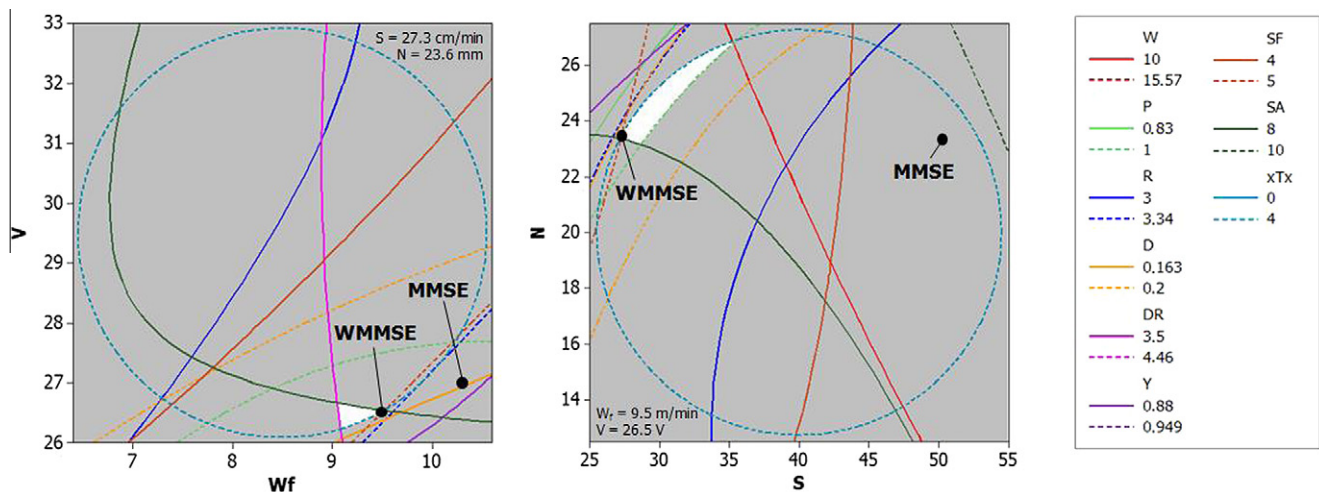


Fig. 12. Overlaid contour graphics.

**Table 13**  
Confirmation experiments.

| Experiment | Geometry       |                |                |       | Productivity        |       | Quality    |            |
|------------|----------------|----------------|----------------|-------|---------------------|-------|------------|------------|
|            | W (millimeter) | P (millimeter) | R (millimeter) | D (%) | DR (kilogram/hours) | Y (%) | SF (score) | SA (score) |
| 1          | 11.07          | 1.04           | 3.47           | 19.8  | 3.57                | 87.0  | 5          | 8          |
| 2          | 11.39          | 0.88           | 3.31           | 19.1  | 3.61                | 87.8  | 5          | 8          |
| 3          | 11.19          | 0.92           | 3.39           | 18.4  | 3.57                | 86.9  | 5          | 8          |
| 4          | 11.35          | 1.02           | 3.42           | 19.5  | 3.60                | 87.7  | 5          | 8          |
| Mean       | 11.25          | 0.96           | 3.40           | 19.2  | 3.59                | 87.3  | 5.0        | 8.0        |
| Prediction | 11.90          | 0.92           | 3.33           | 16.6  | 3.66                | 88.7  | 5.0        | 8.0        |
| Error (%)  | −5.48          | 4.88           | 2.20           | 2.60  | −2.04               | −1.39 | 0.00       | 0.00       |

weight in the new optimization. Nevertheless, the loss in productivity was not significant. This new optimal condition was therefore judged satisfactory.

Fig. 12 illustrates the overlaid contour graphics built for this optimization. Considering the values established as acceptable for each cladding characteristic, these graphics show that the MMSE method was unable to identify an optimal point inside the feasible region. Because of this inability, MMSE was deemed unsatisfactory. The WMMSE, on the other hand, demonstrated just such an ability. Thus, in optimizing this stainless steel cladding process, which considered multiple correlated responses with different degrees of importance, the Weighted Multivariate Mean Square Error showed itself to be a good technique.

## 5. Confirmation experiments

To verify the reproducibility of the results, a series of four confirmation experiments were run with the optimal combination of the welding parameters, i.e.,  $W_f = 9.5$  meter/minutes,  $V = 26.5$  volt,  $S = 27.3$  centimeter/minutes, and  $N = 23.6$  millimeter. Table 13 presents these results and shows that most of the responses presented real values close to the predicted ones. The largest difference, occurring for the bead width, equaled 5.48%. Finally, Fig. 13 shows the optimal weld bead obtained for this stainless steel cladding application. It can be observed here that the WMMSE method successfully conducted the process to a compatible result with the expected objectives.

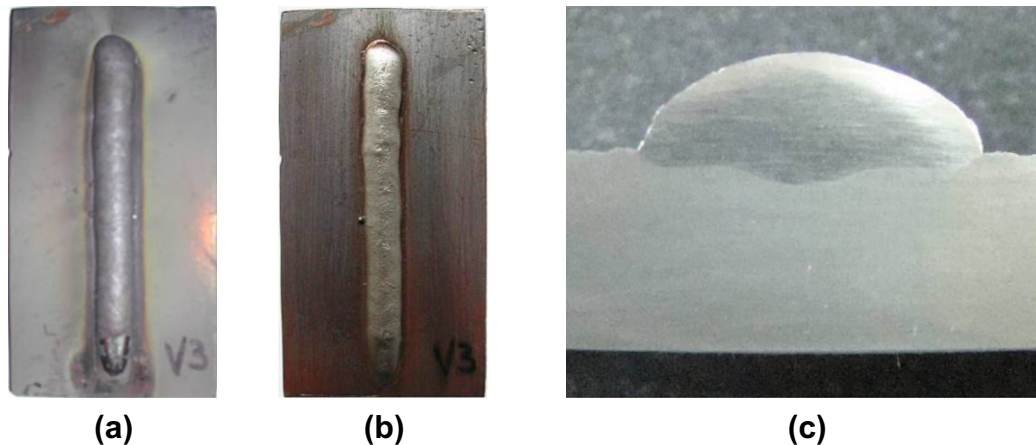


Fig. 13. Optimal weld bead: (a) slag formation, (b) surface appearance, (c) geometry.

## 6. Conclusions

This paper introduced the Weighted Multivariate Mean Square Error, developed to optimize multiple correlated responses presenting different degrees of importance. While capable of dealing with correlated responses, the Multivariate Mean Square Error is unable to attribute them weights. We thus proposed here a weighting procedure, the WMMSE approach. It requires attributing weights before Principal Component Analysis. Furthermore, PCA must be performed taking into account the variance–covariance matrix, so that the principal components absorb the attributed weights. In the WMMSE formulation, the principal components were also weighted, according to their respective degrees of explanation.

We employed a threefold experimental method (RSM, MMSE, and WMMSE) to optimize the flux-cored arc welding used in a stainless steel cladding applications. Response Surface Methodology, applied to model the cladding characteristics, developed reliable mathematical models for the prediction, control, and optimization of the process. All RSM models were statistically significant and most of them presented high adjustments, with  $adj.R^2$  values above 84%. The models' capability of predicting the results was verified by the confirmation experiments; low errors were observed between the theoretical and real values. The correlation analysis identified a moderately correlated structure among the responses, with correlations not too strong but significant. In relation to the Multivariate Mean Square Error, this method presented an optimal weld bead beyond the desired profile. This confirmed the need to optimize the responses with different weights. Thus, the weighting strategy proposed by the Weighted Multivariate Mean Square Error was applied and a feasible optimal point was found. Such results, considering a reliability of 95%, were obtained with the following combination of welding parameters:  $W_f = 9.5$  – meter/minutes,  $V = 26.5$  volt,  $S = 27.3$  centimeter/minutes, and  $N = 23.6$  millimeter.

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## References

- Anjos, M.A., Vilar, R., Qiu, Y.Y., 1997. Laser cladding of ASTM S31254 stainless steel on a plain carbon steel substrate. *Surface & Coatings Technology* 92, 142–149.
- Ardakani, M.K., Noorossana, R., 2008. A new optimization criterion for robust parameter design – the case of target is best. *International Journal of Advanced Manufacturing Technology* 38, 851–859.

- Balasubramanian, V., Lakshminarayanan, A.K., Varahamoorthy, R., Babu, S., 2009a. Application of response surface methodology to prediction of dilution in plasma transferred arc hardfacing of stainless steel on carbon steel. *International Journal of Iron Steel Research* 16, 44–53.
- Balasubramanian, V., Varahamoorthy, R., Ramachandran, C.S., Muralidharan, C., 2009b. Selection of welding process for hardfacing on carbon steels based on quantitative and qualitative factors. *International Journal of Advanced Manufacturing Technology* 40, 887–897.
- Box, G.E.P., Hunter, W.G., MacGregor, J.F., Erjavec, J., 1973. Some problems associated with the analysis of multiresponse data. *Technometrics* 15, 33–51.
- Bratchell, N., 1989. Multivariate response surface modeling by principal components analysis. *Journal of Chemometrics* 3, 579–588.
- Busacca, G.P., Marseguerra, M., Zio, E., 2001. Multiobjective optimization by genetic algorithms: application to safety systems. *Reliability Engineering & System Safety* 72, 59–74.
- Ch'ng, C.K., Quah, S.H., Low, H.C., 2005. Index Cpm in multiple response optimization. *Quality Engineering* 17, 165–171.
- Chiao, H., Hamada, M., 2001. Analyzing experiments with correlated multiple responses. *Journal of Quality Technology* 33, 451–465.
- Derringer, G., Suich, R., 1980. Simultaneous optimization of several response variables. *Journal of Quality Technology* 12, 214–219.
- Georgieva, A., Jordanov, I., 2009. Global optimization based on novel heuristics, low-discrepancy sequences and genetic algorithms. *European Journal of Operational Research* 196, 413–422.
- Govindaluri, S.M., Cho, B.R., 2007. Robust design modeling with correlated quality characteristics using a multicriteria decision framework. *International Journal of Advanced Manufacturing Technology* 32, 423–433.
- He, Z., Zhu, P.F., Park, S.H., 2012. A robust desirability function method for multi-response surface optimization considering model uncertainty. *European Journal of Operational Research* 221, 241–247.
- Huang, M.S., Lin, T.Y., 2008. Simulation of a regression-model and PCA based searching method developed for setting the robust injection molding parameters of multi-quality characteristics. *International Journal of Heat and Mass Transfer* 51, 5828–5837.
- Johnson, R.A., Wichern, D., 2007. *Applied Multivariate Statistical Analysis*, sixth ed. Prentice-Hall, New Jersey.
- Kannan, T., Murugan, N., 2006. Effect of flux cored arc welding process parameters on duplex stainless steel clad quality. *Journal of Materials Processing Technology* 176, 230–239.
- Khuri, A.I., Conlon, M., 1981. Simultaneous optimization of multiple responses represented by polynomial regression functions. *Technometrics* 23, 363–375.
- Köksoy, O., 2006. Multiresponse robust design: Mean Square Error (MSE) criterion. *Applied Mathematics and Computation* 175, 1716–1729.
- Kovach, J., Cho, B.R., 2009. A D-optimal design approach to constrained multiresponse robust design with prioritized mean and variance considerations. *Computers and Industrial Engineering* 57, 237–245.
- Kuo, I.C., Chou, C.P., Tseng, C.F., Lee, I.K., 2009. Submerged arc stainless steel strip cladding – effect of post-weld heat treatment on thermal fatigue resistance. *Journal of Materials Engineering and Performance* 18, 154–161.
- Lin, D.K.J., Tu, W., 1995. Dual response surface optimization. *Journal of Quality Technology* 27, 34–39.
- Liu, S., Moskowitz, H., Plante, R., Preckel, P.V., 2002. Product and process yield estimation with Gaussian quadrature (GQ) reduction: improvements over the GQ full factorial approach. *European Journal of Operational Research* 140, 655–669.
- Majumdar, J.D., Pinkerton, A., Liu, Z., Manna, I., Li, L., 2005. Mechanical and electrochemical properties of multiple-layer diode laser cladding of 316L stainless steel. *Applied Surface Science* 247, 373–377.
- Montgomery, D.C., 2009. *Design and Analysis of Experiments*, sixth ed. John Wiley & Sons, New York.
- Murugan, N., Parmar, R.S., 1994. Effects of MIG process parameters on the geometry of the bead in the automatic surfacing of stainless steel. *Journal of Materials Processing Technology* 41, 381–398.

- Murugan, N., Parmar, R.S., 1997. Stainless steel cladding deposited by automatic gas metal arc welding. *Welding Journal* 76, 391–403.
- Paiva, A.P., Campos, P.H., Ferreira, J.R., Lopes, L.G.D., Paiva, E.J., Balestrassi, P.P., 2012. A multivariate robust parameter design approach for optimization of AISI 52100 hardened steel turning with wiper mixed ceramic tool. *International Journal of Refractory Metals and Hard Materials* 30, 152–163.
- Paiva, A.P., Ferreira, J.R., Balestrassi, P.P., 2007. A multivariate hybrid approach applied to AISI 52100 hardened steel turning optimization. *Journal of Materials Processing Technology* 189, 26–35.
- Paiva, A.P., Paiva, E.J., Ferreira, J.F., Balestrassi, P.P., Costa, S.C., 2009. A multivariate mean square error optimization of AISI 52100 hardened steel turning. *International Journal of Advanced Manufacturing Technology* 43, 631–643.
- Palani, P.K., Murugan, N., 2006. Development of mathematical models for prediction of weld bead geometry in cladding by flux cored arc welding. *International Journal of Advanced Manufacturing Technology* 30, 669–676.
- Palani, P.K., Murugan, N., 2007. Optimization of weld bead geometry for stainless steel claddings deposited by FCAW. *Journal of Materials Processing Technology* 190, 291–299.
- Phillips, A.L., 1965. *Welding Handbook: Special Welding Processes and Cutting*, 5th ed. Macmillan, London.
- Poojari, C.A., Varghese, B., 2008. Genetic Algorithm based technique for solving Chance Constrained Problems. *European Journal of Operational Research* 185, 1128–1154.
- Safizadeh, M.H., 2002. Minimizing the bias and variance of the gradient estimate in RSM simulation studies. *European Journal of Operational Research* 136, 121–135.
- Shahi, A.S., Pandey, S., 2008a. Effect of auxiliary preheating of the filler wire on quality of gas metal arc stainless steel claddings. *Journal of Materials Engineering and Performance* 17, 30–36.
- Shahi, A.S., Pandey, S., 2008b. Modelling of the effects of welding conditions on dilution of stainless steel claddings produced by gas metal arc welding procedures. *Journal of Materials Processing Technology* 196, 339–344.
- Tarn, Y.S., Juang, S.C., Chang, C.H., 2002. The use of grey-based Taguchi methods to determine submerged arc welding process parameters in hardfacing. *Journal of Materials Processing Technology* 128, 1–6.
- Vining, G.G., 1998. A compromise approach to multiresponse optimization. *Journal of Quality Technology* 30, 309–313.